CHIRP SCALING ALGORITHMS FOR SAR PROCESSING

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Abstract - The **chirp** scaling SAR processing algorithm is both accurate and efficient. Successful implementation requires proper selection of the interval of output samples, which is a function of the chirp interval, signal sampling rate, and signal bandwidth. Analysis indicates that for both airborne and spaceborne SAR applications in the slant range domain a linear chirp scaling is suffucient. To perform nonlinear interpolation process such as to output ground range SAR images, one can use a nonlinear chirp scaling interpolater presented in this paper.

Keywords: interpolator, sampling rate, valid samples, SAR processing, range-Doppler spectrum, nonlinear chirp scaling.

CHIRP SCALING INTERPOLATOR

Chirp scaling interpolation [1] is a process that yields almost perfect interpolation result. Although some people refrain from calling it an interpolator for its lack of' resemblance to an interpolator, chirp scaling however functions like an interpolator. Strictly speaking, it only deals with input signal with constant sample spacing and 'it only generates output signal with constant sample spacing. In another word, it only rescales the input signal in the horizontal axis.

The chirp scaling process can be described mathematically as follows. Let $S_1(t)$ be the input and $S_2(t)$ be the output of a chirp scaling interpolator. $S_2(t)$ can be written as

$$S_2(t) = ((((S_1(t) \otimes p_1(t)) \cdot p_2(t)) \otimes p_3(t)) \cdot p_4(t))$$

where $\otimes$ stands for convolution, and $p_1(t), p_2(t), p_3(t)$, and $p_4(t)$ are the chirp signals involved in this scaling process. Furthermore, we express these chirps in the following form:

$$p_1(t) = \exp\{-j2\pi\frac{b_1}{2}t^2\}, \quad p_3(t) = \exp\{j2\pi\frac{(b_1+b_2)}{2}t^2\}$$

$$p_2(t) = \exp\{-j2\pi\frac{b_2}{2}t^2\}, \quad p_4(t) = \exp\{j2\pi\frac{b_2}{b_1}\frac{(b_1+b_2)}{2}t^2\}$$

Expressed in integral form, $S_2(t)$ is given by

$$S_2(t) = \int\int S_1(t-\tau_1-\tau_2)p_1(\tau_1)p_2(t-\tau_2)p_3(\tau_2)\,d\tau_1 d\tau_2\; p_4(t)$$

Let $\tau_1' = \tau_1 + \tau_2$, the above equation is then given by

$$S_2(t) = \int\int S_1(t-\tau_1')p_1(\tau_1'-\tau_2)p_2(t-\tau_2)p_3(\tau_2)\,d\tau_1' d\tau_2\; p_4(t)$$

or $S_2(t) = \int S_1(t-\tau_1^{'}) \exp\{-j2\pi(\frac{b_1}{2}\tau_1^{'2} + \frac{b_2}{2}t^2 - \frac{(b_1+b_2)}{2}\frac{b_2}{b_1}t^2)\}\cdot$

$$d\tau_1^{'} \int \exp\{-j2\pi(b_1\tau_2\tau_1^{'} + b_2\tau_2 t)\}d\tau_2$$

Since the second integral is a delta function, $S_2(t)$ is therefore given by

$$S_2(t) = \int S_1(t-\tau_1^{'}) \exp\{-j2\pi(\frac{b_1}{2}\tau_1^{'2} + \frac{b_2}{2}t^2 - \frac{b_1+b_2}{2}\frac{b_2}{b_1}t^2)\}\cdot$$

$$\delta(b_1\tau_1^{'} + b_2 t)d\tau_1^{'}$$

The delta function implies that when $\tau_1^{'} = -(b_2/b_1)t$, the result of the above integral can be shown to be

$$S_2(t) = S_1(\frac{b_1+b_2}{b_1}t)$$

This proves that the output signal $S_2(t)$ is the input signal $S_1(t)$ scaled by a factor of $(b_1+b_2)/b_1$ along the time axis. If the scaling factor is α, then b_2 is given by $b_1(\alpha-1)$.

To implement this interpolator in discrete form, several factors must be considered due to the limited bandwidth and finite sampling frequency. First, in order to preserve information, one should not increase the sampling spacing to a point below the Nyquist rate of $S_1(t)$. Therefore, if $S_1(t)$ is already sampled at the Nyquist rate, one should use the chirp scaling interpolator only to reduce its sampling spacing. Second, the bandwidth of $p_1(t)$ should be no less than the bandwidth of $S_1(t)$ and no greater than the sampling frequency of $S_1(t)$. In a design to optimize the length of the valid interval of $S_2(t)$, the bandwidth of $p_1(t)$ is set to be the bandwidth of $S_1(t)$. Third, the time intervals for $p_1(t), p_2(t), p_3(t)$, and $p_4(t)$ should be:

$$\mathrm{T}_1 = B/|b_1|, \qquad \mathrm{T}_2 = \mathrm{T}_{in} + \mathrm{T}_1$$

$$\mathrm{T}_3 = \frac{f_s}{b_1+b_2}, \qquad \mathrm{T}_4 = \mathrm{T}_{out}$$

where B is the bandwidth of $S_1(t)$, f_s is the sampling frequency of $S_1(t)$, T_{in} is the time duration of $S_1(t)$, and T_{out} is the time duration of $S_2(t)$. It can be proven (Appendix A) that to prevent degradation to signal resolution, T_{out} must be bounded by

$$\mathrm{T}_{out} = \mathrm{Min}\{\frac{\gamma-\alpha}{|\alpha-1|}\mathrm{T}_1, \mathrm{T}_{in}\}$$

where $\alpha = (b_1 + b_2)/b_1$ and $\gamma = f_s/B$. This indicates that the time interval of the valid output signal is a function of the time interval of the first chirp used in this interpolation process. For $\gamma = 1.18$, the ratio between T_{out} and T_1 as a function of α is depicted in Figure 1. It should be noted that the output sampling interval is given by α/f_s.

For a band-limited system, the weight function of an ideal interpolator must follow a $\sin x/x$ type function. The equivalent weight function of a chirp scaling interpolator is of a slightly different form. The difference in their far sidelobes cause a very slight difference in the magnitude and phase of the output signal. To accurately analyze the phase fidelity performance, these subtle differences cannot be ignored.

AIRBORNE CHIRP SCALING PROCESSING

If the range histories of all the targets in a two dimensional data block are identical, a 2-D fast Fourier correlation can directly be applied to achieve both range and azimuth correlation simultaneously. To accomplish this, we perform a 2-D FFT (an azimuth FFT followed by a range FFT) for the SAR raw data, perform a 2-D FFT on the point-target 2-D impulse response, multiply the first spectrum by the complex conjugate of the second spectrum, then perform a 2-D inverse FFT. For real SAR data, the range history of targets varies in the cross-track dimension. Therefore, the image generated from the above algorithm will be focused only for a very shallow slant range interval in which the range histories match that of the reference function. Above and below that interval, resulting images are smeared in both the range and azimuth directions. The smearing along azimuth is mainly due to the mismatch of the focusing parameters between the phase function of the target impulse response and that of the reference response. The smearing along range is due to the mismatch of the range curvatures between the two responses.

Due to the mismatches in range curvature and focusing parameters, the impulse responses for targets in the cross-track direction when presented in the range-Doppler domain typically look like that given in Figure 2. In this figure, the straight horizontal line at the center is the energy trace of the impulse response corresponding to the matched targets. The phase of each sample along this line is well compensated. Above and below this line, the target spectrum follows a curve, of which the curvature becomes greater for targets further away from the straight line. These curves accounts for the residual curvatures that are left uncompensated by the reference function. As shown in [2], the range spacing between any two adjacent curves is given by $1/\sqrt{1-(\lambda f/2v)^2}$

which depends only on the Doppler frequency f. The phase of each sample along each curve follows that of a FM function with a small time-bandwidth product. These phase functions account for the mis match between the focusing parameters.

A simple fix to the 2-D spectrum describe above is to extract all 1-D azimuth spectra from the curves in the range-Doppler data shown in Figure 2 and to multiply each extracted spectrum with a 1-D phase function to compensate for the residual phase error. After the range-Doppler spectrum is fixed, a well focused image can be obtained by applying an inverse azimuth FFT to this data. It is obvious that the chirp scaling interpolator can be applied to extract the desired spectra. In fact, the algorithm reported in [2,3] is to combine the simplified 2-D processing with the chirp scaling process.

In a combined process, we may consider that the chirp scaling process is to be accomplished by four sets of chirps, $p_1(\tau,\omega)$, $p_2(\tau,\omega)$, $p_3(\tau,\omega)$, and $p_4(\tau,\omega)$, where τ is the time coordinate of range and ω is the angular frequency in azimuth. Since most of the radar employ a linear FM chirp, the process of convolving the signal with $P_1(\tau,\omega)$ can be avoided. In addition, the chirp rate $b_1(\omega)$ of P_1 is inherently determined by the radar pulse as K in the (τ,t) domain and $K'(\omega)$ in the (τ,ω) domain due to the effect of the azimuth impulse response [2]. Let the range scaling factor be $\alpha(\omega)$, the chirp rate of P_2, P_3, and P_4 in (τ,ω) are therefore given by $(\alpha(\omega)-1)K'(\omega)$, $\alpha(\omega)K'(\omega)$, and $\alpha(\omega)(\alpha(\omega)-1)K'(\omega)$. However, P_3 is actually implemented in (ω_r,ω) domain, therefore, its corresponding chirp rate is given in the form of reciprocal, $1/(\alpha(\omega)K)$. The removal of "$\prime$" in K is due to the multiplication of the 2-D reference spectrum which removes the effect of the azimuth impulse response on p_1.

To be further noted is that the time origin for applying p_2 must follow the range migration curve of the reference target. Therefore, $p_2(\tau,\omega)$ is given by

$$p_2(\tau,\ \omega) = \exp\{-j2\pi \frac{(\alpha(\omega)-1)K'(\omega)}{2}(\tau - \tau_{ref}(\omega))^2\}$$

In applying p_4 to the SAR data, compensation has been made to the range migration of the reference target, the time origin for p_4 is therefore given by a constant of $(2r_{ref})/c$.

AIRBORNE SAR SCALE FACTOR

To derive the scale factor used in SAR processing, one need to derive the slant range history as a function of the minimum slant range and the Doppler frequency. According to the geometry of an aircraft SAR as shown in Figure 3, the slant range at time t is given by $r(t, r_0) = \sqrt{r_0^2 + v^2 t^2}$, where r_0 is the minimum slant range and v is the aircraft velocity. $r(t, r_0)$ can also be expressed by the squint angle θ_s

$$r(t, r_0) = r_0 / \sin\theta_s$$

Since the Doppler frequency at squint angle θ_s is given by $f = 2v\cos\theta_s$. The slant range history as a function of Doppler is, therefore, given by

$$r(f, r_0) = r_0 / \sqrt{1 - (\frac{\lambda}{2} \cdot \frac{f}{v})^2}.$$

The scaling factor is then given by

$$f_{sc} = \frac{dr(f, r_0)}{dr_0} = 1 / \sqrt{1 - (\frac{\lambda}{2} \cdot \frac{f}{v})^2}$$

SPACEBORNE SAR SCALE FACTOR

For a spaceborne SAR with a narrow beamwidth, the slant range history can be formulated by its Doppler frequency f_d and frequency rate f_r

$$r(t, r_0) = r_0 - \frac{\lambda}{2} f_d(r_0) t - \frac{\lambda}{4} f_r(r_0) t^2$$

Since the Doppler frequency f and time are related by $f = f_d(r_0) + t \cdot f_r(r_0)$, the slant range function can be rewritten as

$$r(f, r_0) = r_0 - \frac{\lambda}{4} \frac{f^2 - f_d^2(r_0)}{f_r(r_0)}$$

Assuming a circular polar orbit over the equator of a spherical surface, the $f_d(r_0)$ and $f_r(r_0)$ can be expressed by

$$f_d(r_0) = \frac{2V_e \sin\theta_e}{\lambda}$$

$$f_r(r_0) = \frac{2(r_0 A \cos\theta_L - v_s^2)}{\lambda r_0^2} + \frac{2V_e^2 \sin^2\theta_i}{\lambda r_0}$$

where A is the spacecraft acceleration, V_e is the earth rotation velocity, θ_L, and θ_i are the look angle and incidence angle, respectively. It can be shown that

$$\theta_i = \cos^{-1}\frac{R_0^2 + R_e^2 - R_b^2}{2R_0 R_e}, \quad \theta_L = \cos^{-1}\frac{R_0^2 + R_b^2 - R_e^2}{2R_0 R_b}$$

where R_e is the radius of earth and R_b is the radius of the orbit. Let $r_0 = R_{ref} + r_1$, the scaling factor can be obtained as

$$f_{sc} = \frac{dr(f, r_0)}{dr_1}$$

A Taylor's series expansion of this scale factor can be made to check the linearity of the scale factor as a function of r_1. The numerical results indicate that the second order coefficient is too small to be considered.

NONLINEAR CHIRP SCALING INTERPOLATOR

The existing chirp scaling processing can only be applied to SAR processing in the slant range domain due to its constraint of linearity. However, for most SAR applications, it is desirable to present the SAR images in the ground range. In such cases, post interpolation is still required to convert slant range image into ground range image. In the following analysis, it will show that the chirp scaling interpolator can be modified to allow it handle nonlinear interpolation problems. By combining this nonlinear chirp scaling interpolator into SAR correlation process, ground range images can be obtained with both high geometric accuracy and high processing efficiency.

Lets consider an interpolator that generates samples having sample spacing being a linear function of time. Again, using the same mathematical process as described in (1). However, the waveforms of $p_2(t)$ and $p_3(t)$ are chosen as follows:

$$p_2(t) = \exp\{-j2\pi ct^3\}, \; p_3(t) = \exp\{j2\pi \frac{b_1}{2} t^2\}$$

It can be shown that

$$S_2(t) = \int S_1(t - \tau_1') \exp\{-j2\pi \frac{b_1}{2} \tau_1'^2\} \delta'(b_1\tau_1' + b_2 t) d\tau_1' p_4(t),$$

where $\delta'(\tau) = \int \exp\{-j2\pi(\tau\tau_2 + 3ct\tau_2^2 - c\tau_2^3)\} d\tau_2$. For c being small enough $\delta'(\tau)$ can be approximated by the delta function, therefore, the following result is obtained.

$$S_2(t) = S_1(t + \frac{ct^2}{b_1}) \exp\{-j2\pi \frac{b_1}{2} (\frac{ct^2}{b_1})^2\} p_4(t)$$

By letting $p_4(t) = \exp\{j2\pi \frac{b_1}{2} (\frac{ct^2}{b_1})^2\}$, we have

$$S_2(t) = S_1(t + \frac{ct^2}{b_1}) = S_1(t + \rho t^2).$$

In implementation, several factors must be considered. The first is that aliasing effect should not be observed for any valid output samples. The second is that the third and second order phase terms in δ' should not change the resolution and pixel location of δ' by more than a set of prescribed upper bounds. To prevent aliasing, the time interval must be bounded according to

$$2\pi 3c(\frac{T}{2})^2 \leq f_s - B$$

To maintain a resolution broadening being less than 10% of the 3-dB width, the coefficient c must be bounded by [4]

$$2\pi 3c(\frac{T}{2})\frac{T_1}{2}^2 \leq 1.82$$

where T_1 is the interval of p_1 (f). To maintain location offset being less than 10% of the 3-dB width, the coefficient c must be bounded by [4]

$$2\pi c\frac{T_1}{2}^3 \leq 0.58$$

In a nonlinear interpolator design, the time bandwidth product, TBP, for $p_1(t)$ is determined first, the time interval T_1 is then given by TBP$/B$, where B is the bandwidth of the input signal. Therefore, b_1 is given by B/T_1, c is given by ρb_1, and T is obtained from aliasing constraint equation.

CONCLUSION

The previously reported chirp scaling SAR processing algorithm incorperated a linear chirp scaling inter polation process. Detailed analysis on the interval of valid output samples are given in this paper. The scale factor analysis indicates that this algorithm is suitable for both airborne and spaceborne SAR applications in the slant range domain. To output ground range SAR images, a nonlinear interpolator must be used, This paper finally presents an efficient and accurate nonlinear chirp scaling interpolator as a candidate for solving that problem.

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APPENDIX A

Let the input signal be given in the interval of $[0, T_{in}]$. The input signal can be considered as the superposition of $T_{in} \cdot f_s$ delta functions with its amplitude given by the amplitude of the signal at each time sample. After convolving with the first chirp $p_1(t)$, each delta function becomes a chirp with a frequency interval of $[0, B]$. After multiplied by the second chirp $p_2(t)$, each chirp function becomes a chirp with a frequency range of $[f_1(t), f_1(t) + \alpha B]$, where $f_1(t)$ is given by the product of the time of the sample in S_1 and the chirp rate $b_1 + b_2$.

To prevent aliasing, $f_1(t)$ must be greater than or equal to 0 and $f_1(t) + \alpha B$ must be less than or equal to f_s. Let t_1 be the time such that $f_1(t_1) = 0$ and t_2 be the time such that $f_1(t_2) + \alpha B = f_s$, then T_{out} is given by $t_2 - t_1$ or

$$T_{out} = \frac{f_s - \alpha B}{|b_2|} = \frac{\gamma - \alpha}{|\alpha - 1|} T_1$$

Since T_{out} is also bounded by T_{in}, therefore

$$T_{out} = \mathrm{Min}\{\frac{\gamma - \alpha}{|\alpha - 1|} T_1,\ T_{in}\}$$

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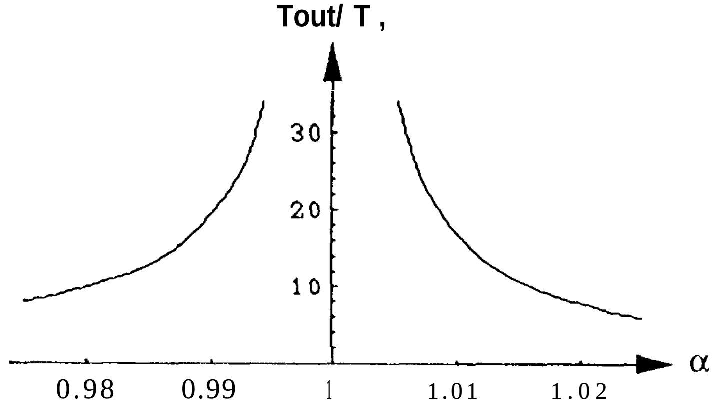

Figure 1. Ratio between thevalid output interval to the interval of the chirp

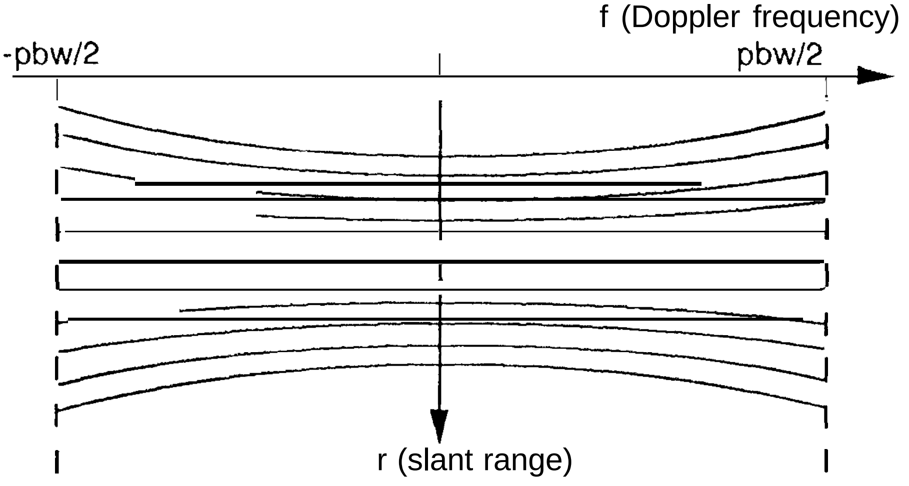

Figure 2. The energy trace of the impulse response spectra from the simple 2-D fast Fourier correlation

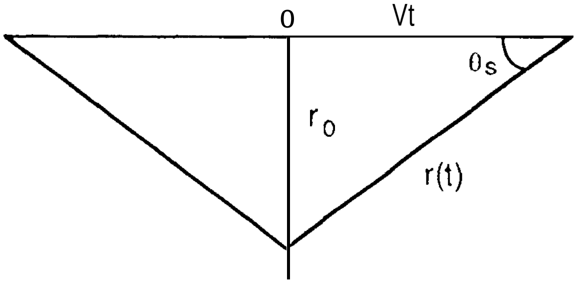

Figure 3. Airborne SAR Geometry